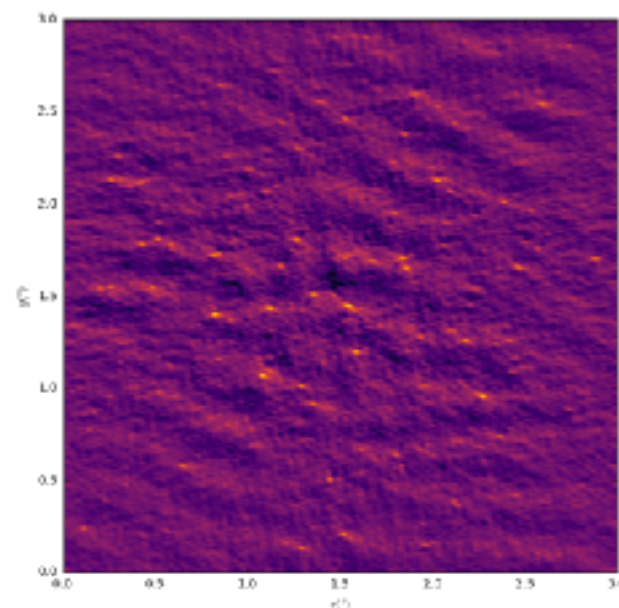


Attempting to measure the power spectrum of radio anisotropies by quadratic estimator

Radio synchrotron background workshop
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Radio power spectrum

- Scientific purpose, motivation
- Method and procedure
- Testing with simulated data
- Preliminary result from real data
- Next steps



• Next steps

Scientific Purposes and Motivations

- Complementary to radio synchrotron isotropy (monopole)
- Detect and trace the anisotropic extragalactic synchrotron emissions (in GHz from 0.1 arcmin to 0.1 deg)
- The Cosmic Web in synchrotron
- Understand how radio emission fits into structure formation
- Use tools from CMB and CIB anisotropy studies
- Use likelihood approach with quadratic estimators
- The well-known radio/FIR correlation means we know roughly what to expect
- Develop a general-purpose tool for estimating power spectra from radio interferometric data

Scientific Purposes and Motivations

Regular approach of measuring the power spectrum include:

- Observe and measure visibility data
- Produce radio images
- Compute correlation function
- Fourier power spectrum

Scientific Purposes and Motivations

Alternatively, radio visibility data is 'already' measured in Fourier domain, i.e. uv-space. The (angular) power spectrum is also defined in Fourier space.

Therefore it worth trying to directly extract the radio power spectrum from the measured visibility data, at least without having to construct radio images.

Consequently it reduce the error propagation with less steps and increase the performance of interferometric data (will delete it in my own word)

Method and Procedures

- Grid the uv-visibility plane
- Apply the quadratic estimator
- Construct dirty map (not necessary)

- Noise estimation
- Attempt with single pointing measurement



Model the sky angular power spectrum

Assume the sky is Gaussian random field with power spectrum

$$\left\langle \tilde{T}(\mathbf{u}) \tilde{T}^*(\mathbf{u}') \right\rangle = C(u) \delta^2(\mathbf{u} - \mathbf{u}')$$

- $\tilde{T}(\mathbf{u})$ is the Fourier transform of the radio sky temperature $T(\mathbf{x})$.
- The relation of u and multipole number l is:

$$l + 1/2 = 2\pi u$$

And the $C(u)$ is related to C_l by:

$$C(u) = C_{2\pi u} \approx C_l$$

(Myers, 2003)

Measured visibility and radio image

- The antennas keep tracking at the phase centre \mathbf{x}_k
- The perceived visibility is the Fourier transform of the sky temperature convolved with the primary beam

$$V_i^k = \int d^2x A(\mathbf{x} - \mathbf{x}_k) e^{-2\pi i \mathbf{u}_i \cdot (\mathbf{x} - \mathbf{x}_k)} T(\mathbf{x}) ,$$

Define the (inverse) Fourier transform of the primary beam and sky temperature as follows (Flat sky approximation):

$$A(\mathbf{x}) = \int d^2u \tilde{A}(\mathbf{u}) e^{2\pi i \mathbf{u} \cdot \mathbf{x}}$$

$$T(\mathbf{x}) = \int d^2u \tilde{T}(\mathbf{u}) e^{2\pi i \mathbf{u} \cdot \mathbf{x}}$$

(Myers, 2003)

Measured visibility and radio image

Obtain the expression of visibility as an integration in uv-space:

$$V_i^k = \int d^2u \tilde{A}(\mathbf{u}_i - \mathbf{u}) e^{2\pi i \mathbf{u} \cdot \mathbf{x}_k} \tilde{T}(\mathbf{u})$$

Discretize the above integration in to summation and formulate as a matrix-vector operation

$$\mathbf{v} = \mathbf{B}\mathbf{t} + \mathbf{n}$$

Each component stores the sky temperature at one grid position.

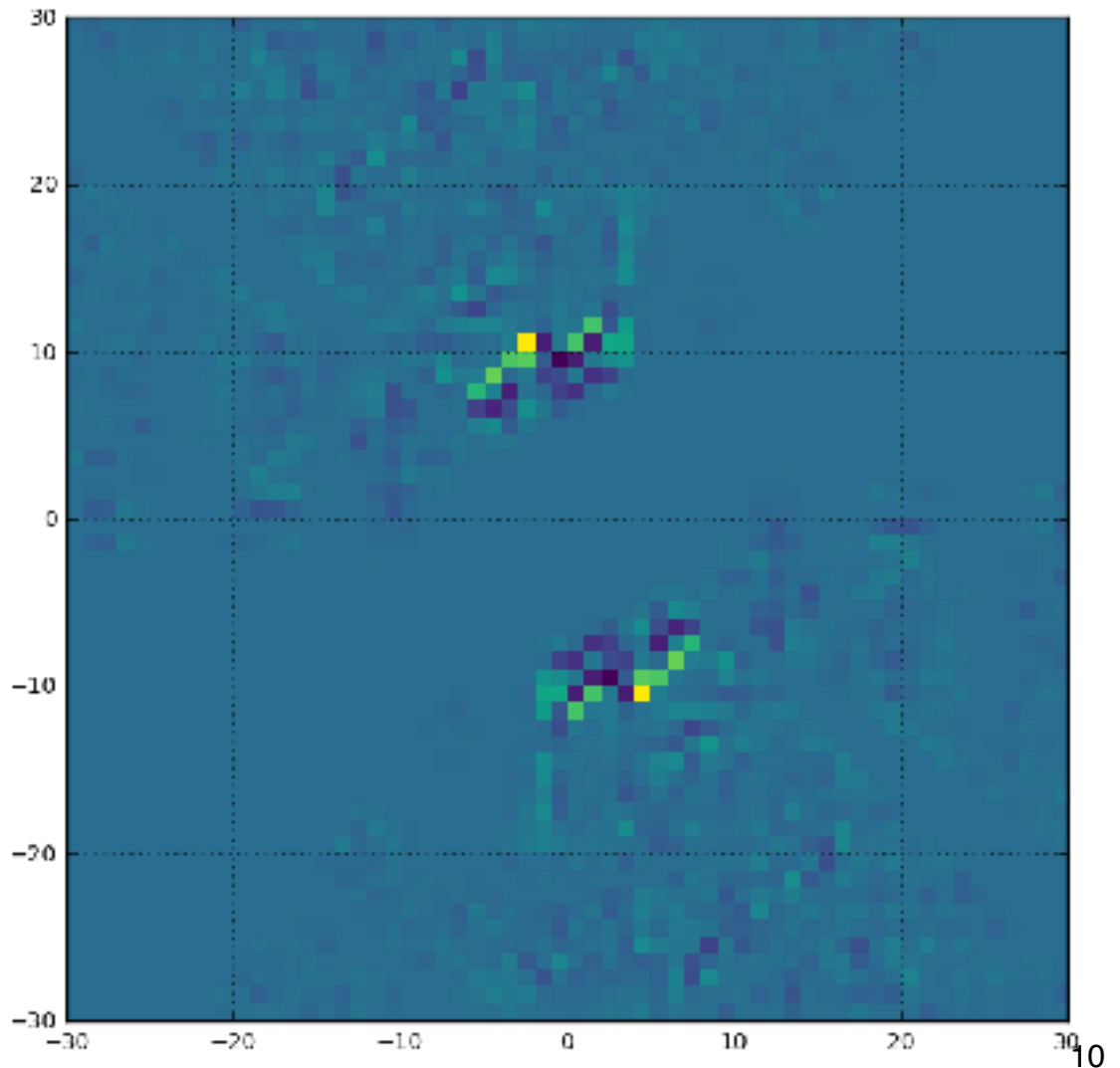
B is the response matrix which transforms the sky temperature into visibility data:

$$[\mathbf{B}]_{(kij)} = \tilde{A}(\mathbf{u}_i - \mathbf{u}_j) e^{2\pi i \mathbf{u}_j \cdot \mathbf{x}_k} \quad (\text{Myers, 2003})$$

Gridded temperature plane

$$T(\mathbf{u})$$

- $T(\mathbf{u})$ has same number of grids (pixels) as the radio image $T(\mathbf{x})$
- is not the quantity measured directly
- is the quantity related to power spectrum



Modelling the power spectrum

Each component of visibility vector is the visibility measurement at one frequency, at a particular uv position with antenna pointing at chosen phase centre x_k .

$$\mathbf{v} = \mathbf{B}\mathbf{t} + \mathbf{n}$$

Recall the definition of power spectrum:

$$\langle \tilde{T}(\mathbf{u})\tilde{T}^*(\mathbf{u}') \rangle = C(u)\delta^2(\mathbf{u} - \mathbf{u}')$$

The matrix version of the power spectrum is:

$$\langle \mathbf{t} \mathbf{t}^\dagger \rangle = \sum_a p_a \mathbf{P}_a$$

(Tegmark, 1997)

Here the power spectrum is binned into different band power. p_a represents fluctuation amplitude of the angular mode. The sets of p_a are the parameters going to be estimated.



Modelling the power spectrum

$$\langle \mathbf{t} \mathbf{t}^\dagger \rangle = \sum_a p_a \mathbf{P}_a$$
$$[\mathbf{P}_a]_{ij} = \begin{cases} 1 & i = j \text{ and } u_{a-1} < |\mathbf{u}_i| < u_a \\ 0 & \text{otherwise} \end{cases}$$

The total visibility data covariance is:

$$\mathbf{C} = \langle \mathbf{v} \mathbf{v}^\dagger \rangle$$

Assume the noise is independent and uncorrelated with the signal, the data covariance matrix is expressed as:

$$\begin{aligned} \mathbf{C} &= \mathbf{B} \langle \mathbf{t} \mathbf{t}^\dagger \rangle \mathbf{B}^\dagger + \langle \mathbf{n} \mathbf{n}^\dagger \rangle \\ &= \sum_a p_a \mathbf{B} \mathbf{P}_a \mathbf{B}^\dagger + \mathbf{N} \\ &= \sum_a p_a \mathbf{C}_a + \mathbf{N} \end{aligned}$$

(Tegmark, 1997)

$$\mathbf{C}_a = \mathbf{B} \mathbf{P}_a \mathbf{B}^\dagger$$

Quadratic Estimator

Employ the quadratic estimator framework for p_a by

$$\hat{p}_a = \sum_b M_{ab} (\hat{q}_b - b_b)$$
$$\hat{q}_a = \frac{1}{2} \mathbf{v}^\dagger \mathbf{E}_a \mathbf{v}$$

The matrix \mathbf{E}_a are undetermined. They group the pairs of visibility data with different weight to estimate the band power.

If the visibility data is Gaussian, the covariance of the power spectrum parameter is:

$$\text{Cov}(\hat{p}_a, \hat{p}_b) = \frac{1}{2} \sum_{cd} M_{ac} M_{bd} \text{Tr} [\mathbf{E}_c \mathbf{C} \mathbf{E}_d \mathbf{C}]$$

(Tegmark, 1997)

Quadratic estimator, constraints

Want the estimator to be unbiased:

$$\langle \hat{p}_a \rangle = p_a$$

Therefore:

$$b_a = \frac{1}{2} \text{Tr} [\mathbf{E}_a \mathbf{N}]$$

$$M_{ab} \text{Tr} [\mathbf{E}_b \mathbf{C}_b] = 2\delta_{ab}$$

(Tegmark, 1997)

Quadratic estimator constraints

Want the parameters are estimated with minimum variance, the optimal solution correspond to inverse variance weighting.

$$\mathbf{E}_a = \mathbf{C}^{-1} \mathbf{C}_a \mathbf{C}^{-1}$$

Recall the definition of the Fisher matrix (with zero mean):

$$F_{ab} = \frac{1}{2} \text{Tr} [\mathbf{C}_a \mathbf{C}^{-1} \mathbf{C}_b \mathbf{C}^{-1}]$$

With inverse variance weighted solution, the mixing matrix M is the inverse of Fisher matrix:

$$M_{ab} = F_{ab}^{-1}$$

(Tegmark, 1997)

Quadratic estimator procedure

Consequently the parameter covariance also equals to the inverse of Fisher matrix.

$$\text{Cov}(\hat{p}_a, \hat{p}_b) = F_{ab}^{-1}$$

All it needs is an initial fiducial model for power spectrum and data covariance matrix to approximate the Fisher matrix and then update the Fisher matrix and band power parameter iteratively

(Tegmark, 1997)

Dirty map (not necessary)

- Image reconstruction is not critical in this approach. The idea of reconstruction is to recover the radio image from visibility data, i.e. invert the previous equation:

$$\mathbf{v} = \mathbf{B}\mathbf{t} + \mathbf{n}$$

- The maximum likelihood of sky estimate turns out to be the Wiener filter solution

$$\hat{\mathbf{t}} = (\langle \mathbf{t} \mathbf{t}^\dagger \rangle^{-1} + \mathbf{B}^\dagger \mathbf{N}^{-1} \mathbf{B})^{-1} \mathbf{B}^\dagger \mathbf{N}^{-1} \mathbf{v}$$

- However if the sky covariance is not known in prior, the exact solution cannot be calculated. As an approximation, keep the numerator only.

$$\hat{\mathbf{t}} = \mathbf{B}^\dagger \mathbf{N}^{-1} \mathbf{v}$$

(Seljak, 1997)

- This dirty map equation only works for a consistency check.

Data: ATCA & SKA simulate skies

- Australia Telescope Compact Array (ATCA)
- ELAIS-S1 region centres at RA=1.4h / Dec = -43 deg
- Frequencies: from 1.08 GHz to 2.09 GHz, 64 channels
- uv-spacing: from 30m to 350m
- 5 antennas in operation

Integration time = 10 seconds

Observation time = 10 hours

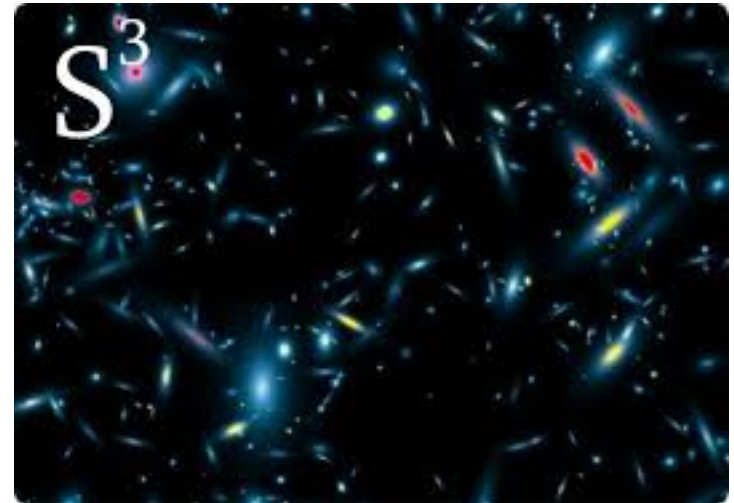
Assumptions: identical aperture, Gaussian primary beam, Gaussian random field, and Gaussian error.



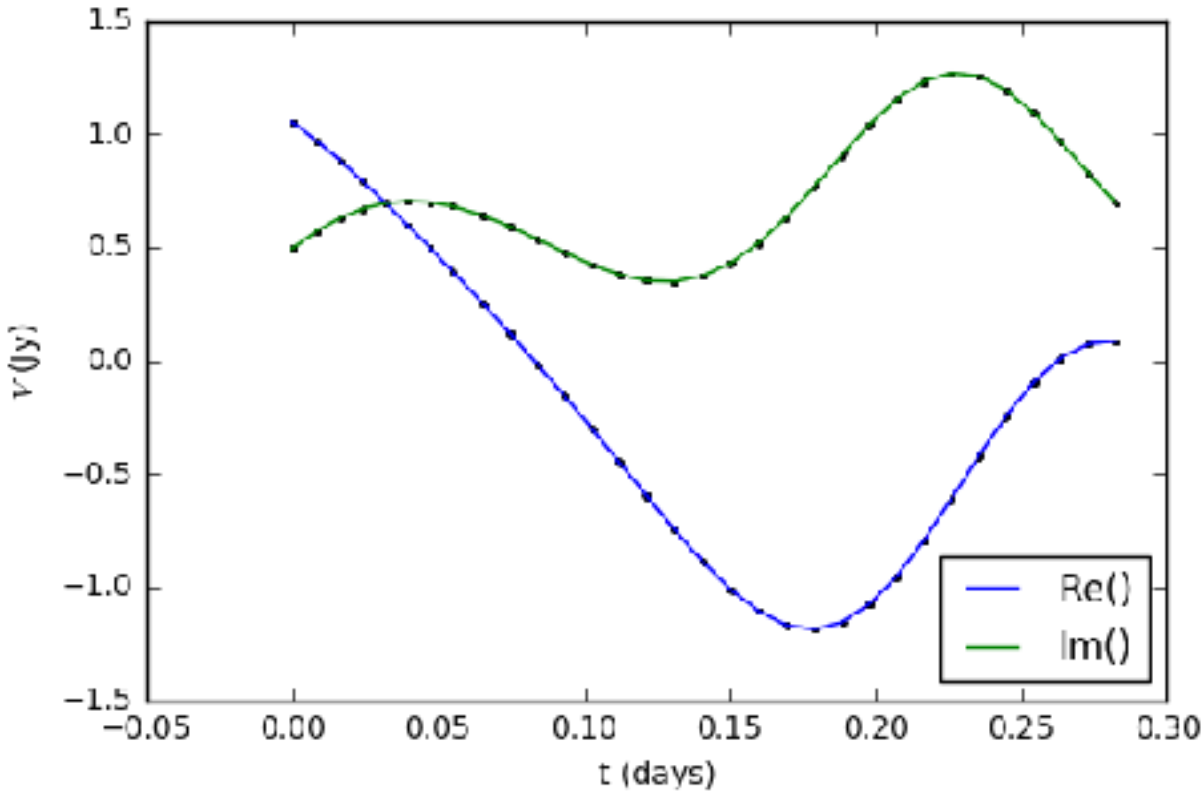
ATCA

Data: ATCA & SKA simulate skies

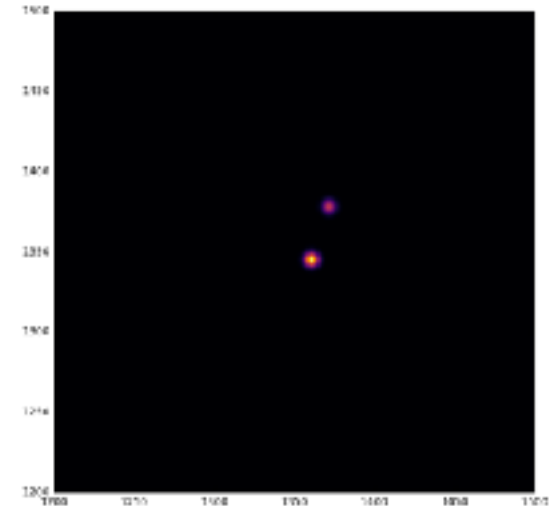
- SKA simulated sky S³ simulation at 1.4GHz (Wilman,2008)
- Use the flux density distribution from the simulated point sources
- Create multifrequency data by single spectral index -0.7
- Distribute the position of the sources randomly or clustered.
- Add extended / diffuse emission feature by convolving Gaussian in arcmin scale.



ATCA Visibility measurement



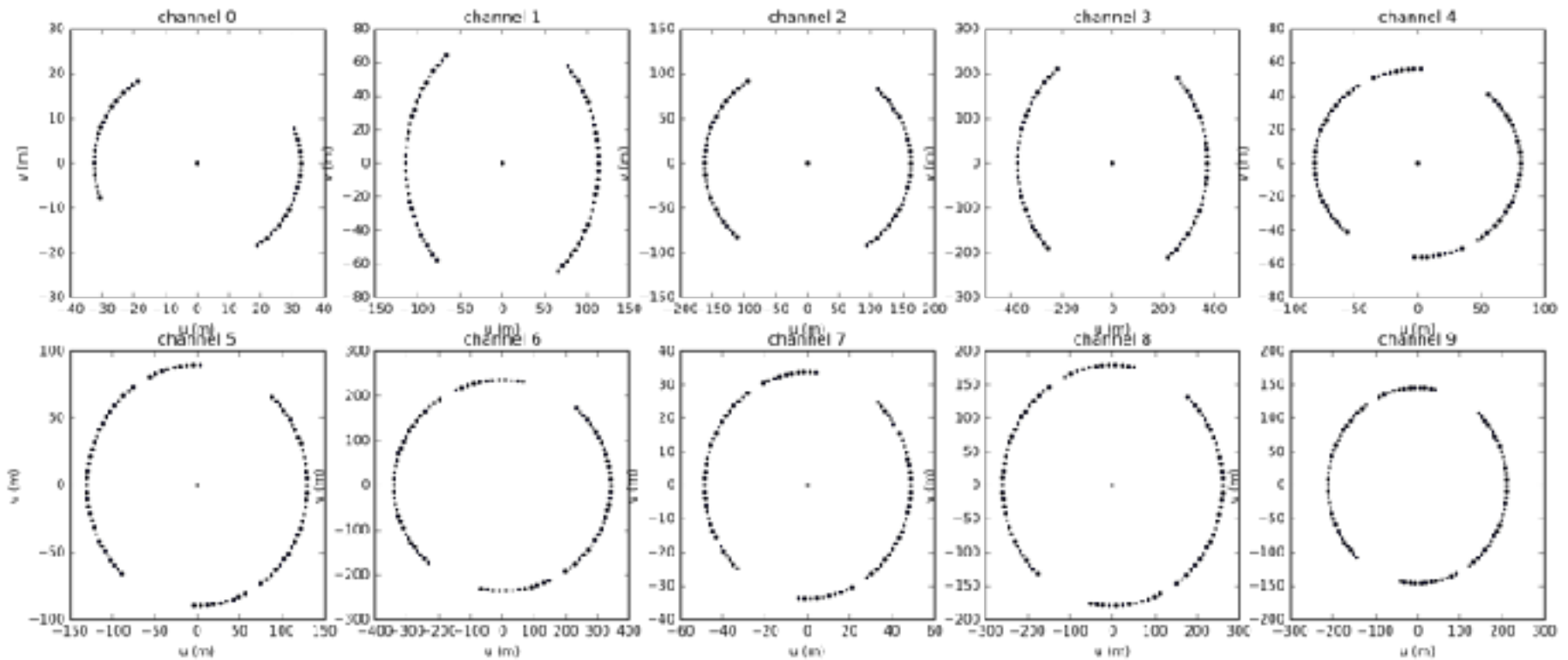
Measurement of one pair of antennas, at one frequency, during the observation time.



Simulation of two point sources

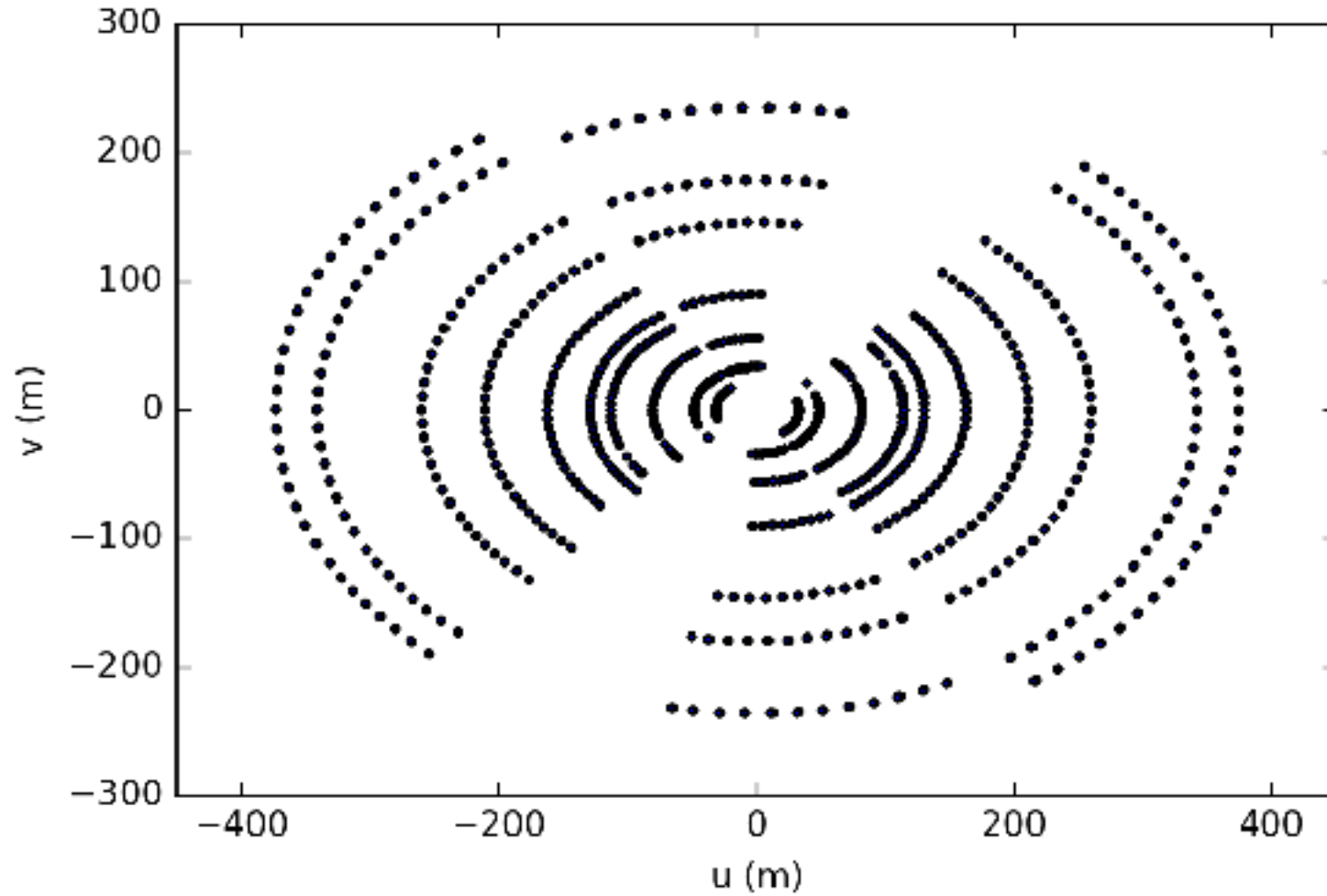
ATCA uv coverage

All baselines at a single frequency

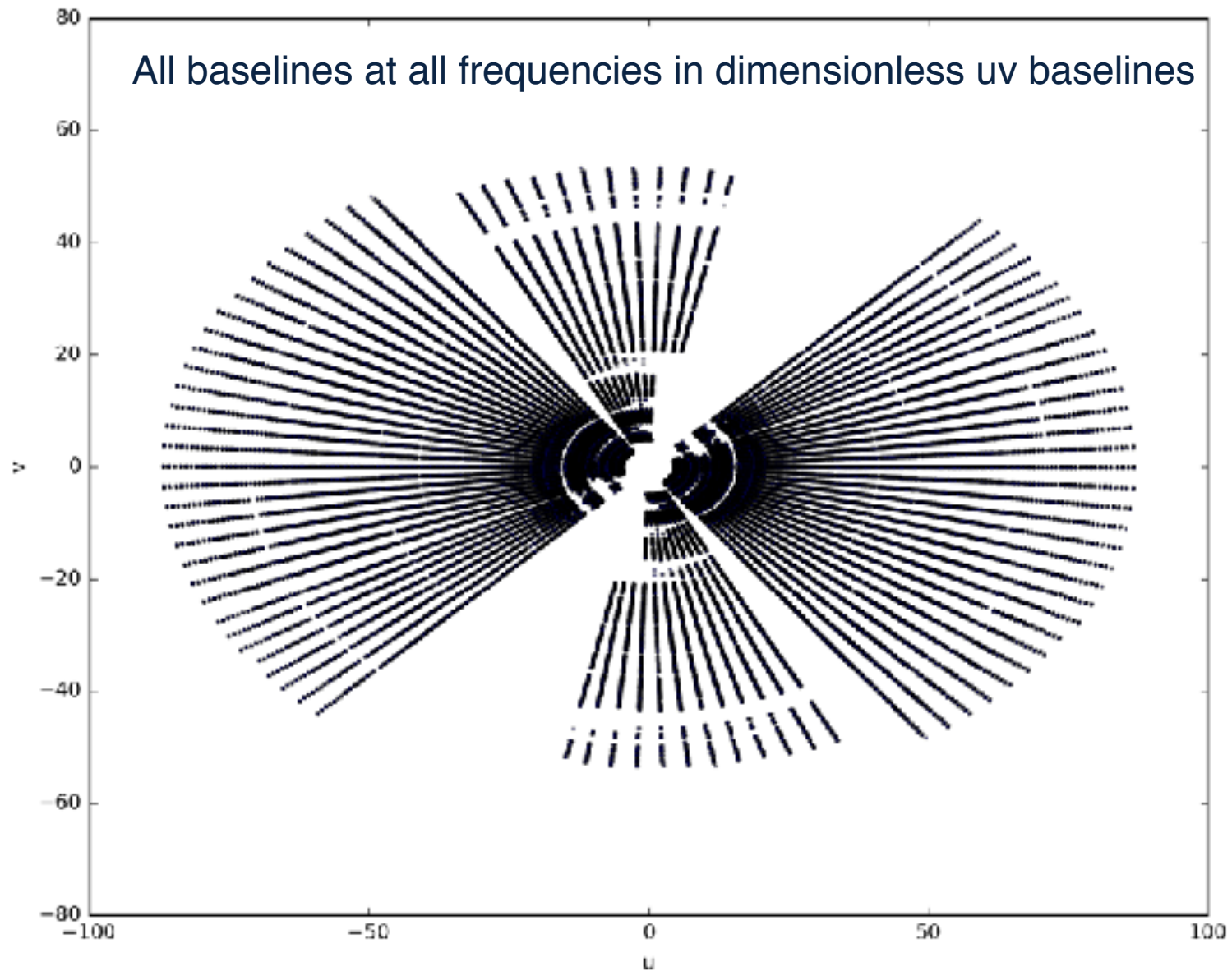


ATCA uv coverage

All baselines at a single frequency



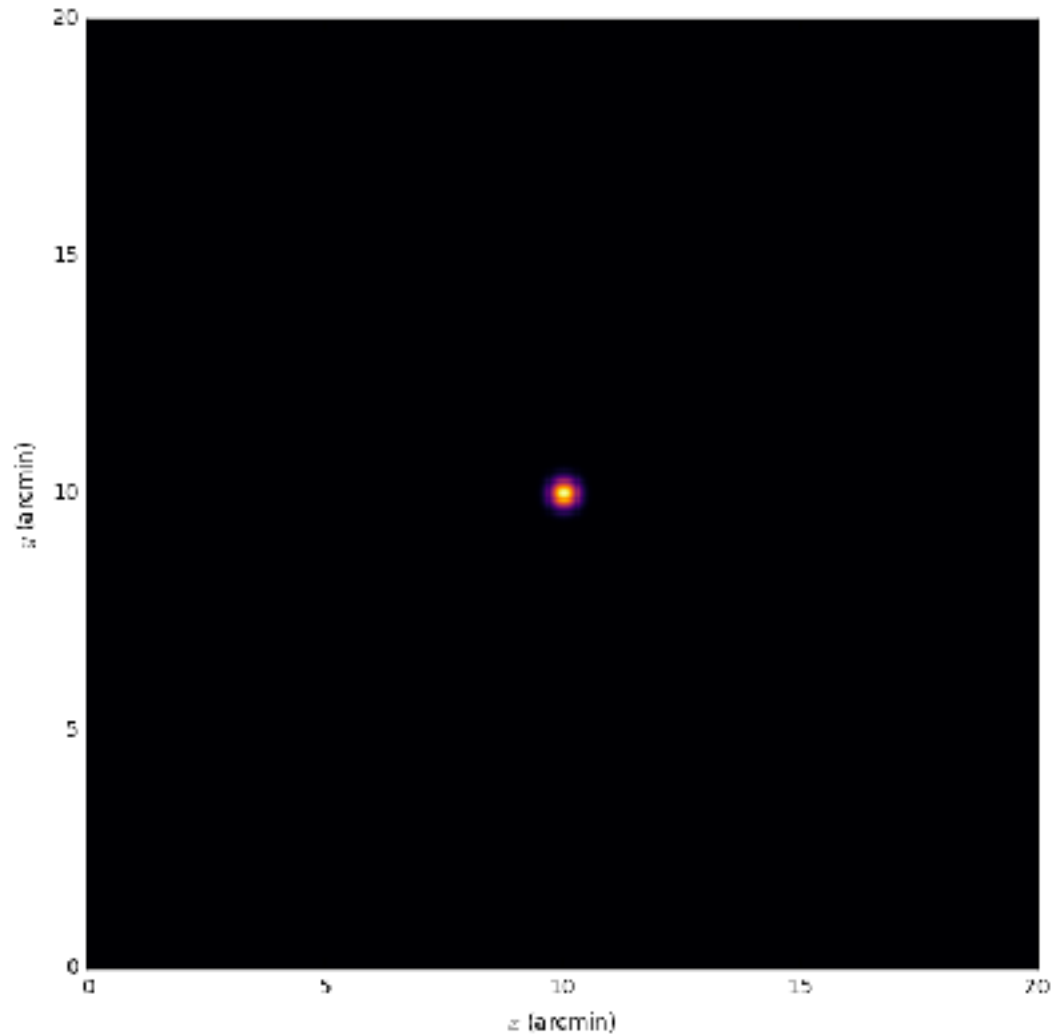
ATCA uv coverage



Checks for a point source image

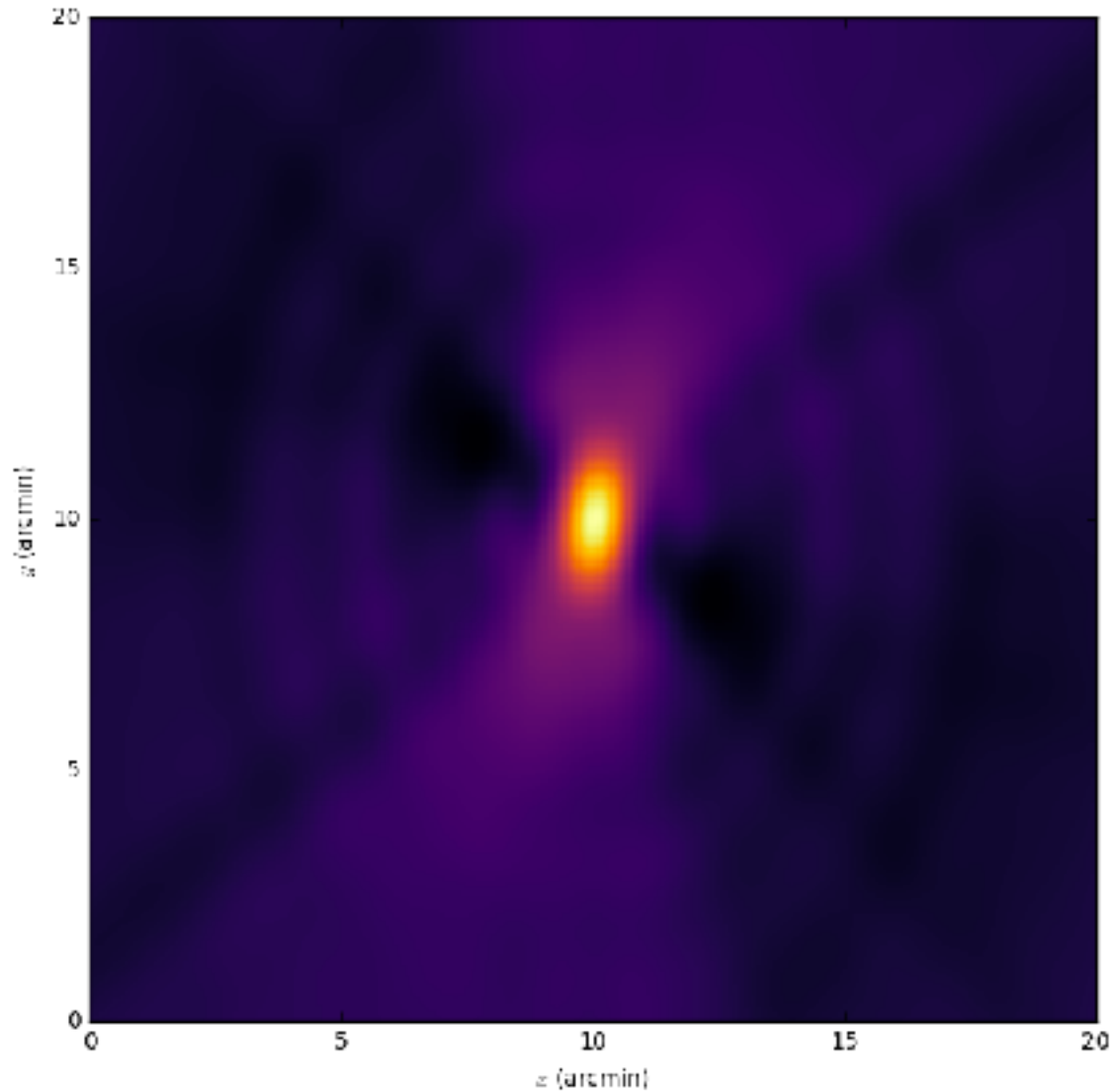
- Simulate a point source image
- Calculate the 'measured' visibility

- Dirty map
- Power spectrum

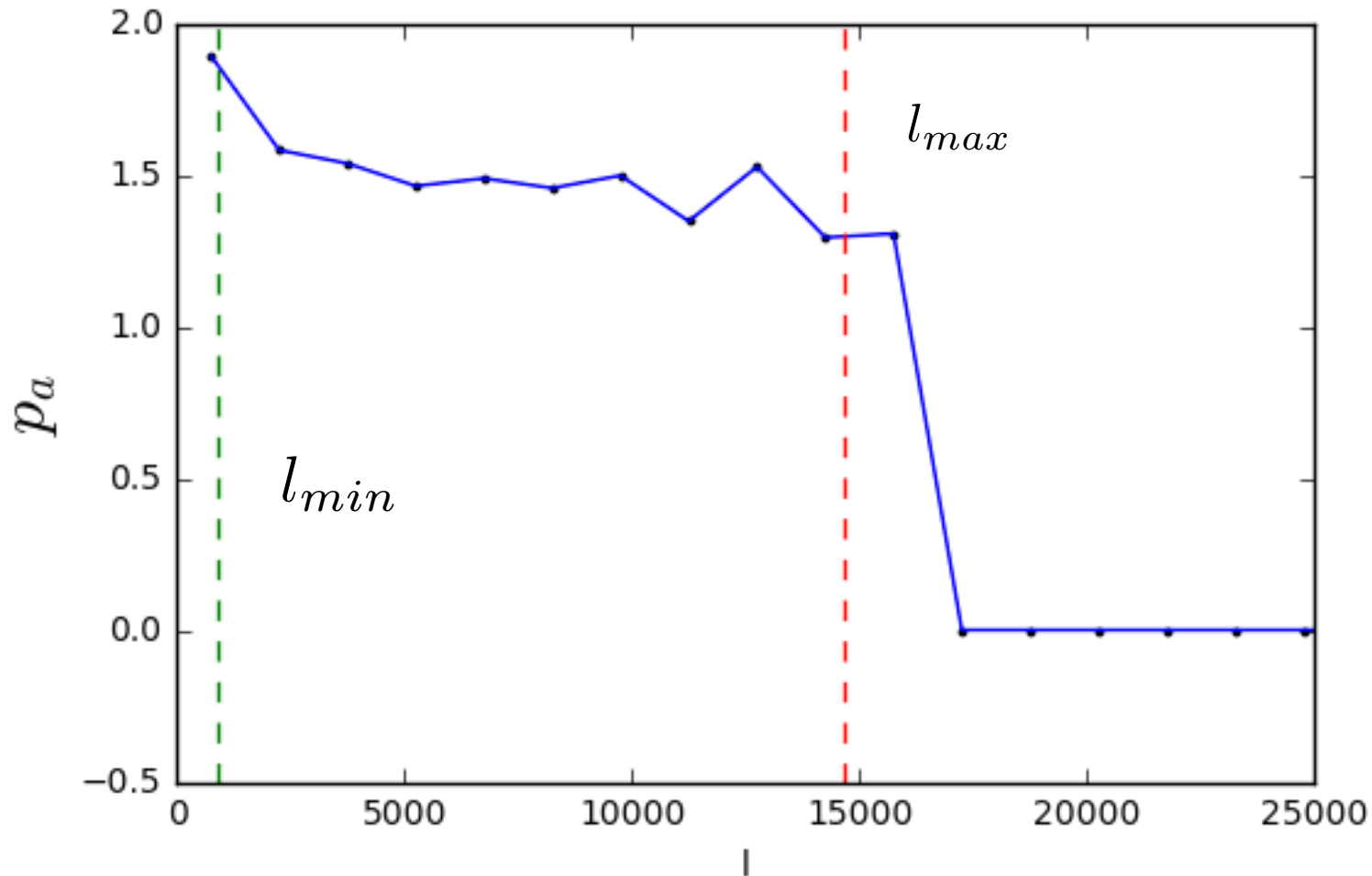


Checks for a point source image

Dirty map: point spread function



Point source power spectrum



$$l = 2\pi|u|$$

Result from simulated sky

A realistic simulation to practice

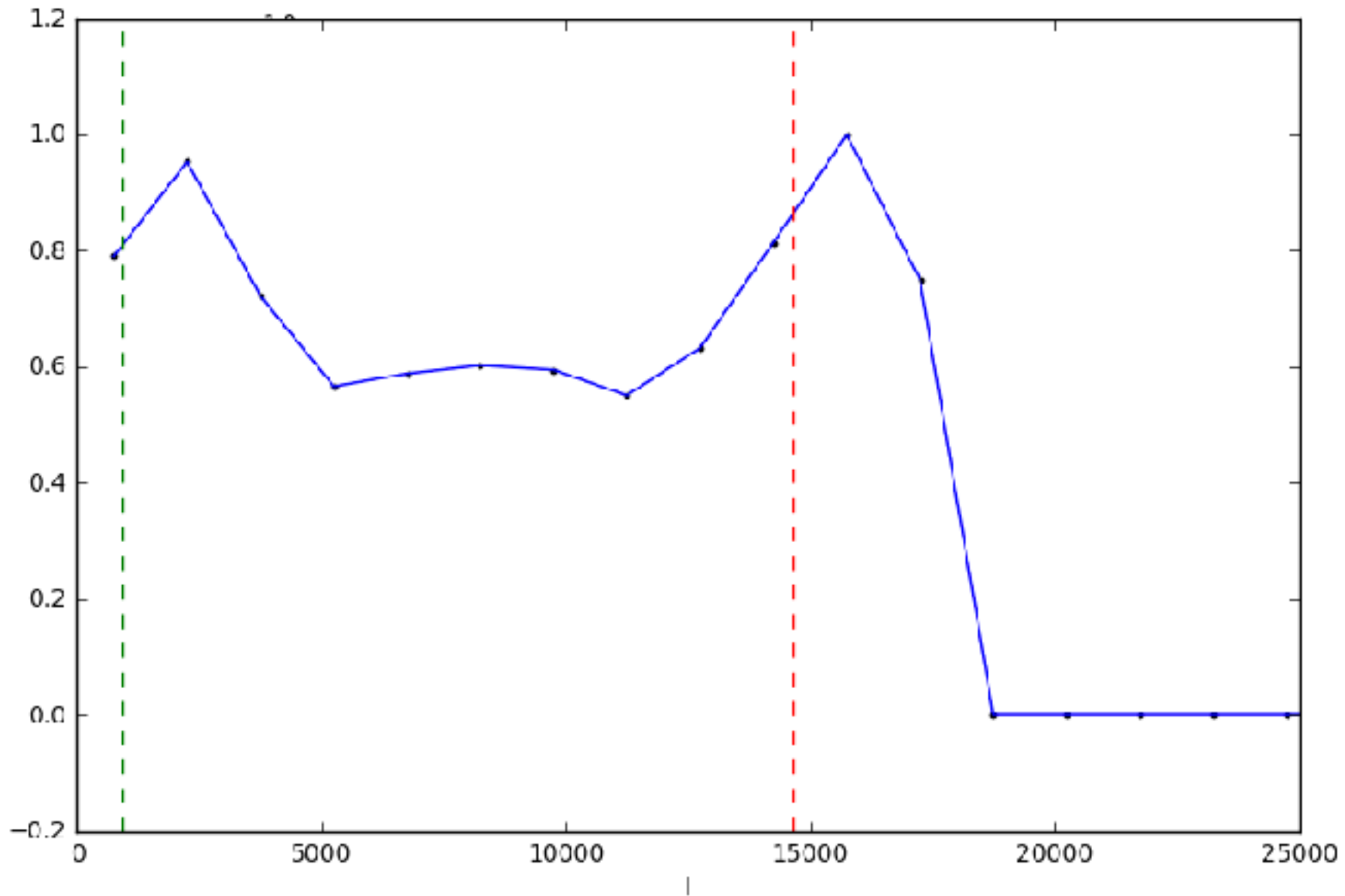
Determine if clustering can be detected (even in principle).



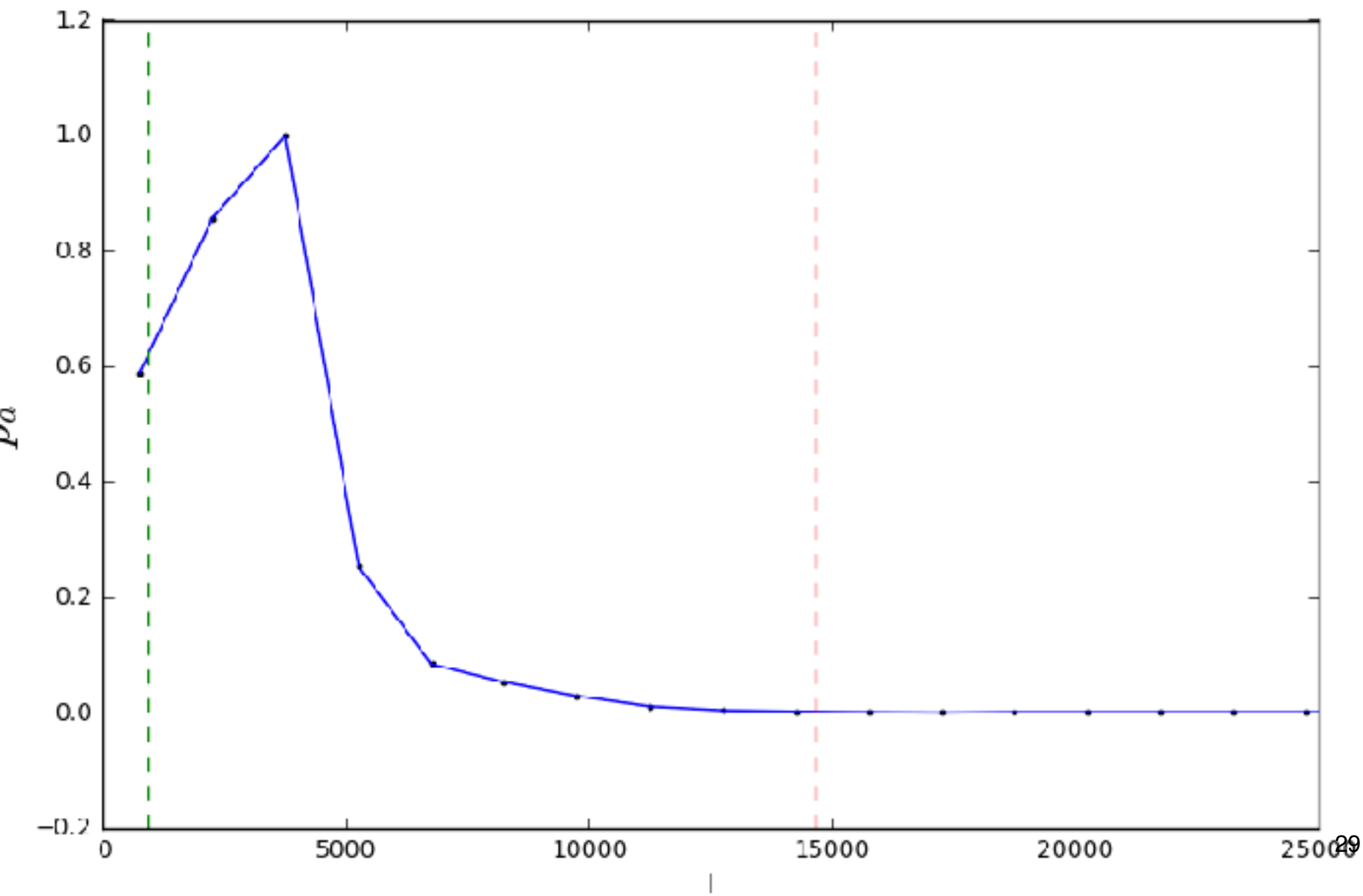
Can calculate power spectrum for

- both point sources + extended emission
 - point sources only
 - extended emission only
-
- Point sources can be distributed either randomly or clustered

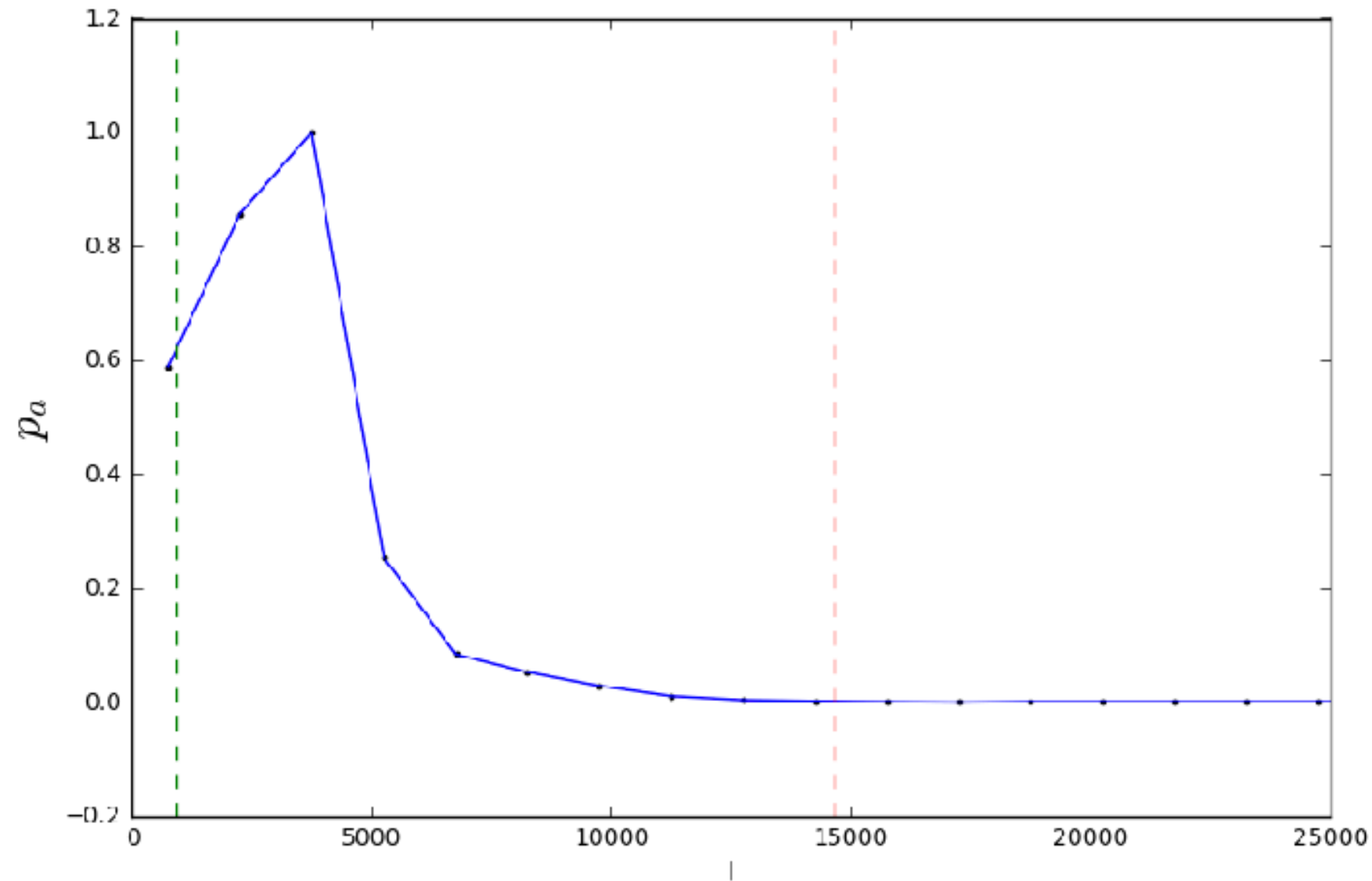
Poisson random point sources only



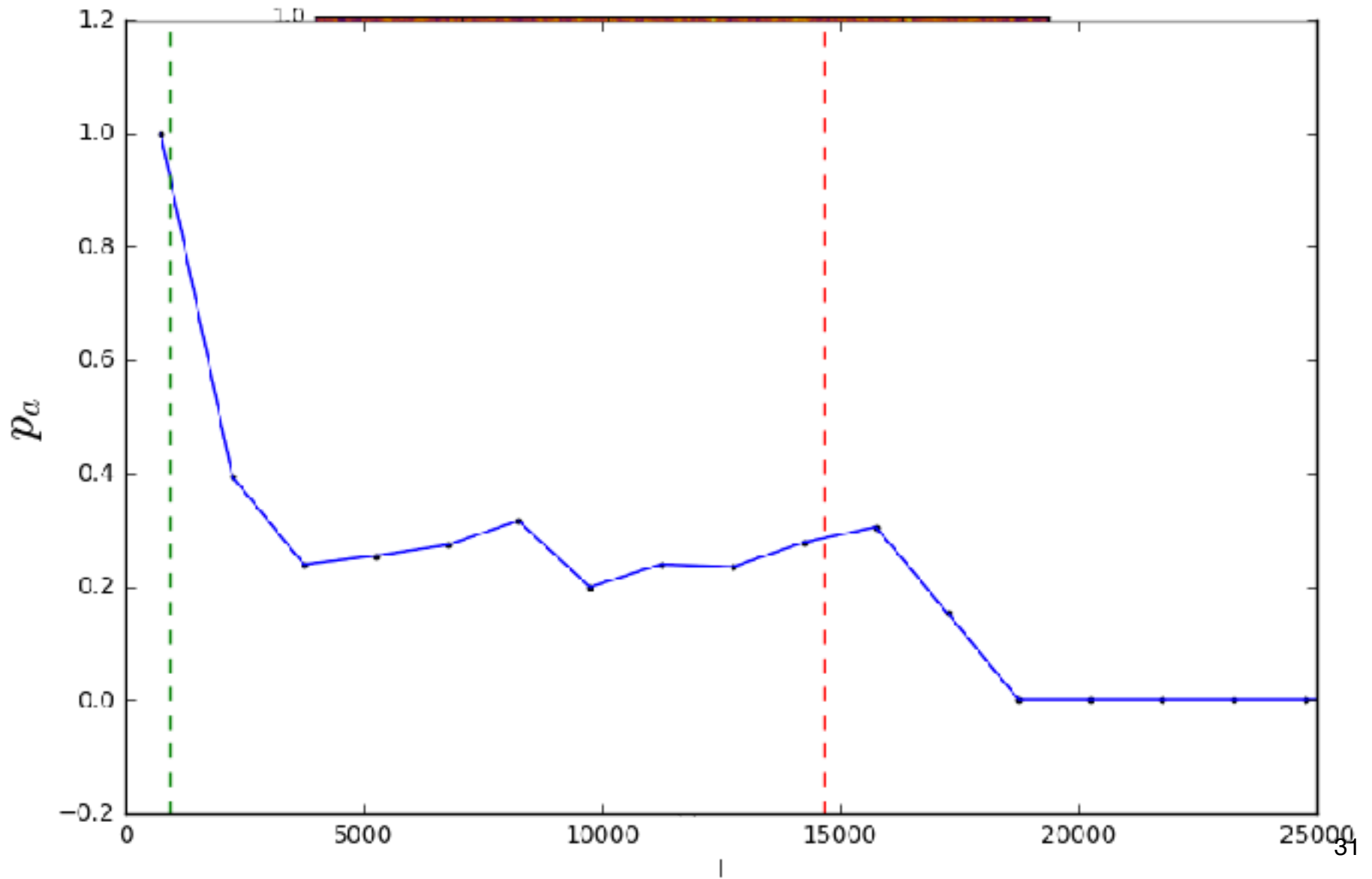
Extended emission only



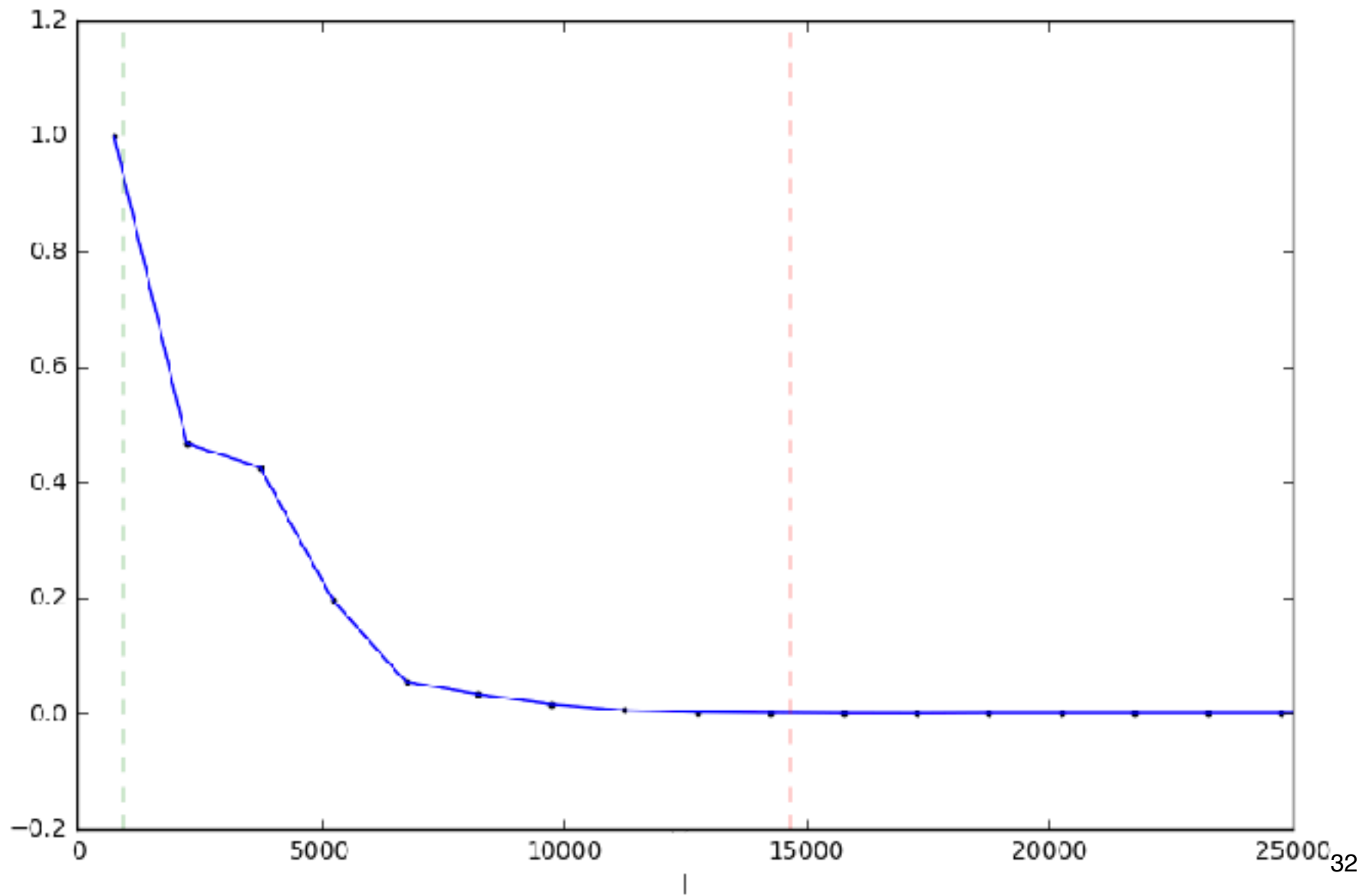
Poisson point sources + extended emission



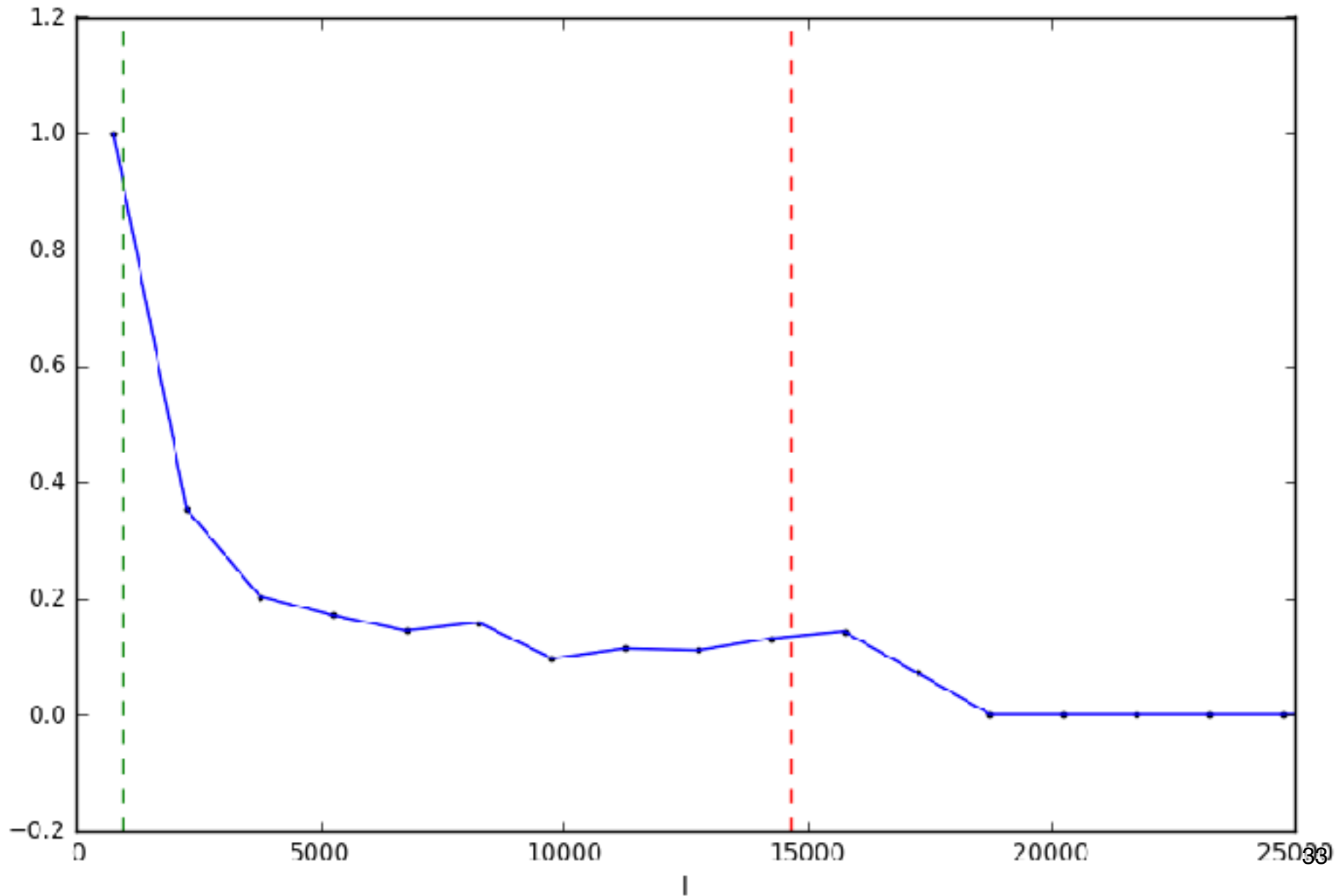
Clustered point sources only



Extended emission only



Clustered point sources + extended emission



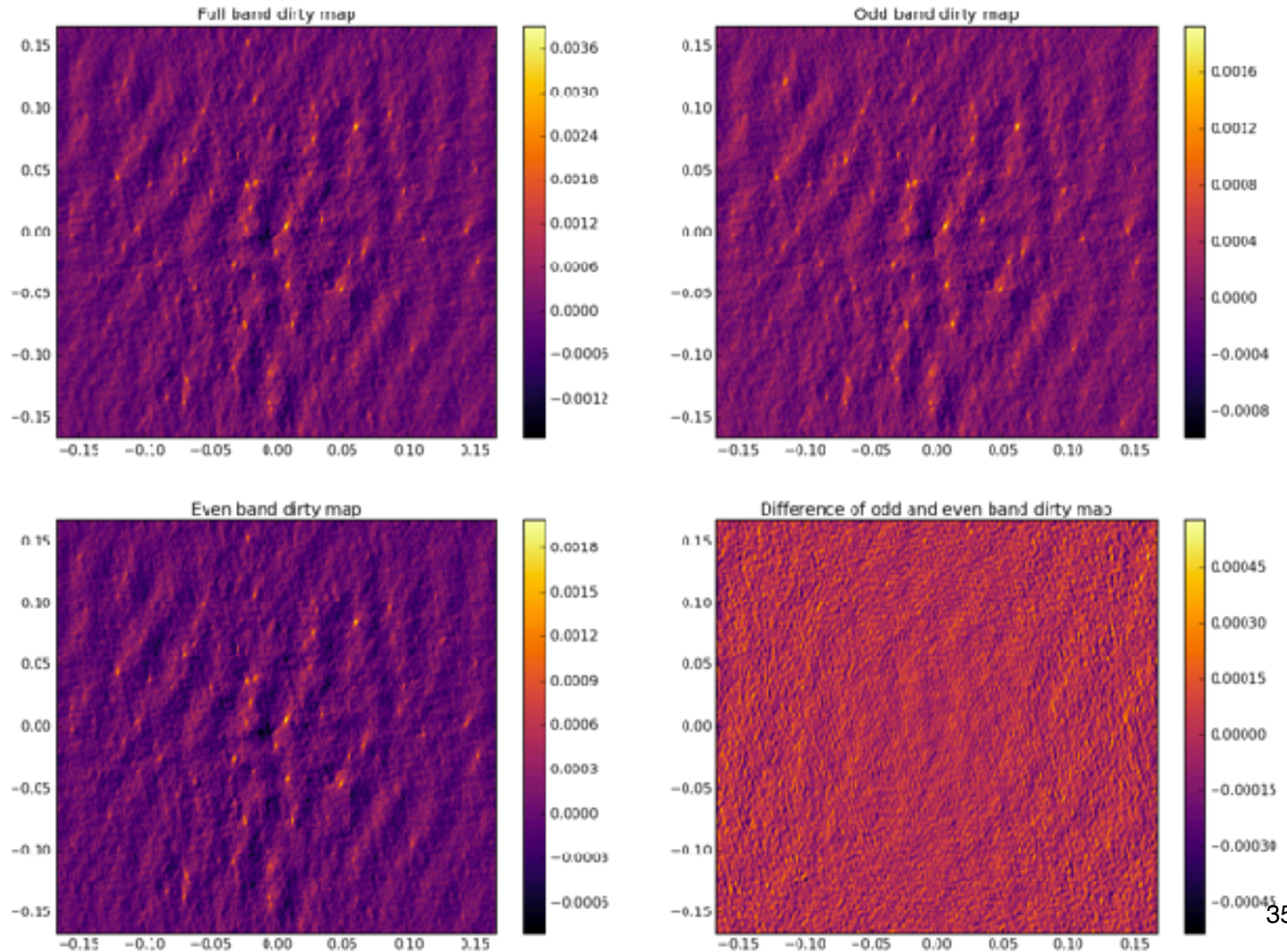
Noise estimation: null test

- Split the data into even and odd frequency
- Split the data into even and odd time measurement
- Monte Carlo simulation
- Stokes V

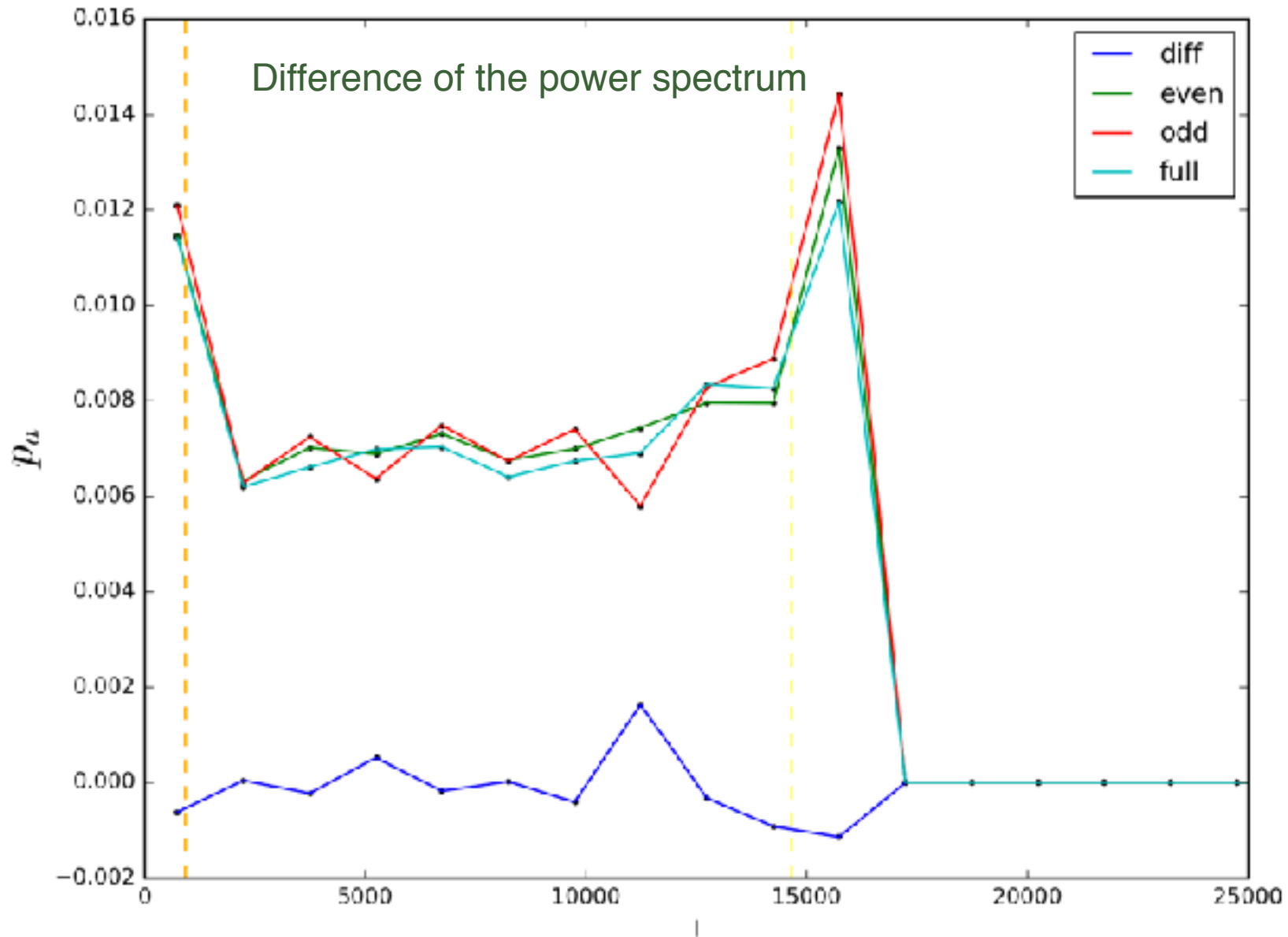
Important for auto-power spectrum estimation



Null test: split in frequency bands

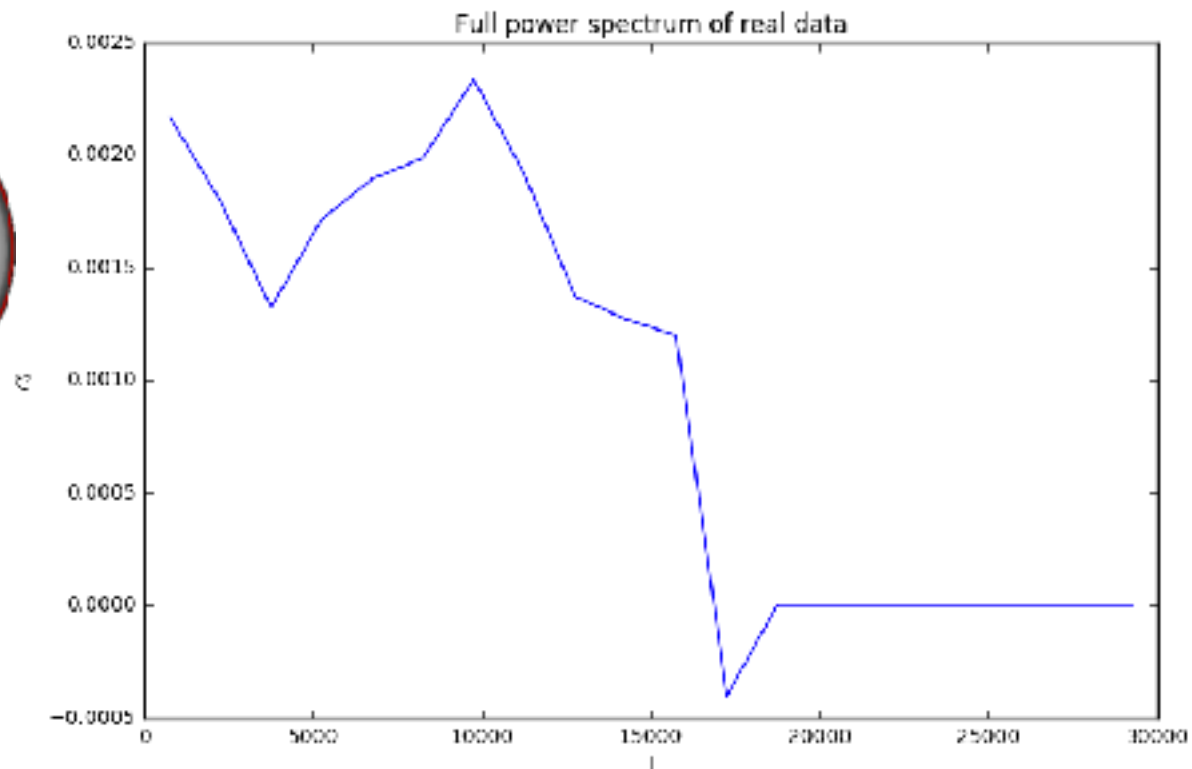
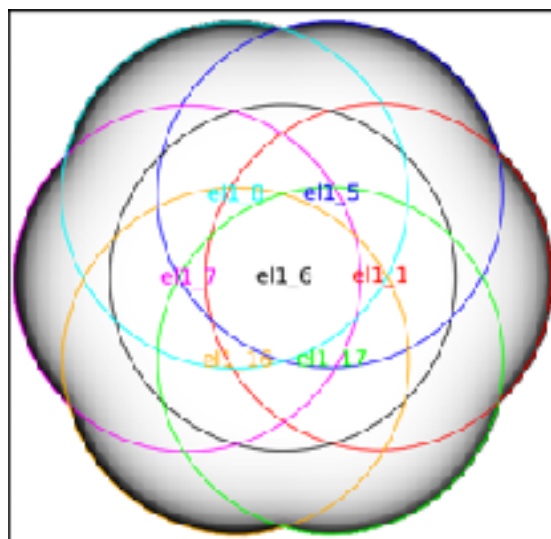


Null test power spectrum



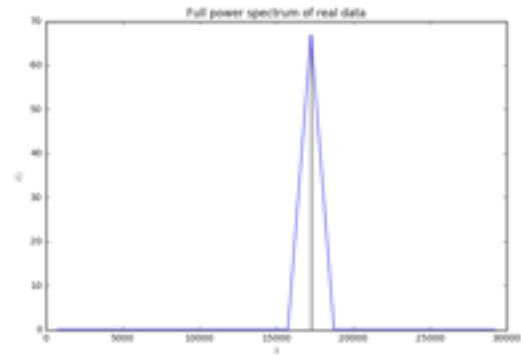
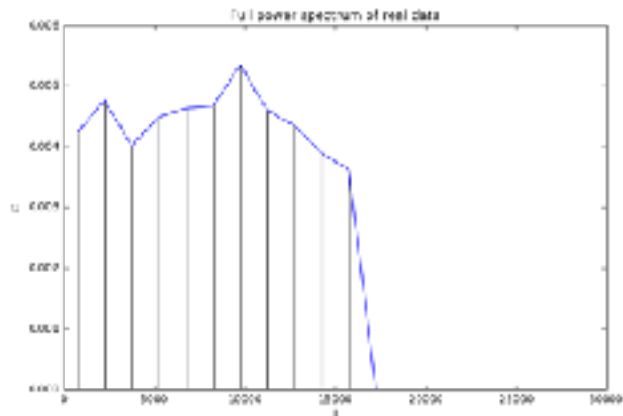
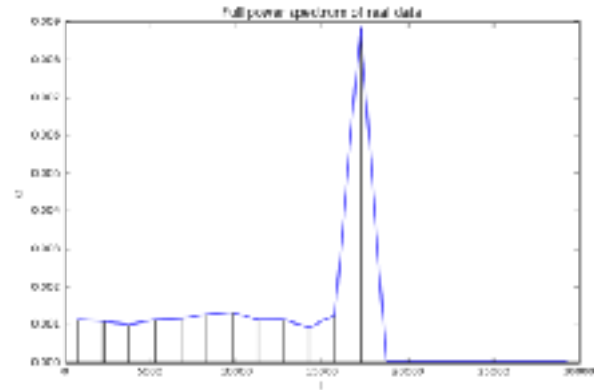
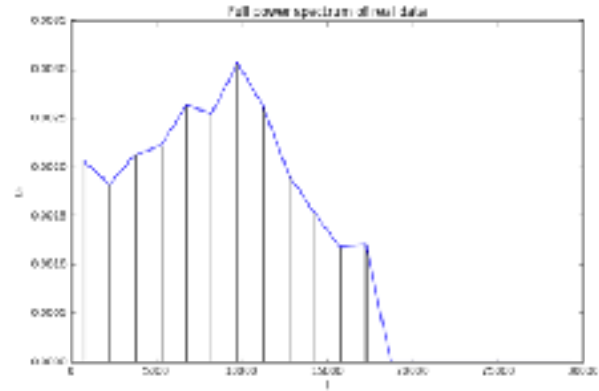
Preliminary results from ATCA real visibility data

- Seven sets of visibility data forming a hexagonal field of view



I am still checking the reliability of the results

The other six



Next steps

- Careful analysis of noise, errors
- Implementation of mosaicking
- Testing with more sizeable data: VLA
- Expecting more suitable observation

